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IFAC PapersOnLine 50-1 (2017) 3685–3690

Nonlinear Robust Control of Atkinson Cycle Engine

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Abstract: In this research, a second order sliding mode (SOSM) strategy to control the Atkinson cycle engine breathing subsystem with an alternative late intake valve closing (LIVC) scheme is presented. The control framework based on extended mean value engine model (EMVEM) of the Atkinson cycle engine in view of fuel economy is evaluated for the standard Federal Urban Driving Schedule (FUDS) and US06 driving cycles. In this context, the authors have already proposed a control-oriented EMVEM model of the Atkinson cycle engine with variable intake valve actuation, well suited for SOSM design. To demonstrate the benefits of the VCR Atkinson cycle engine over conventional Otto cycle engine, in the presence of auxiliary loads and uncertain road loads, its EMVEM model is simulated by using a precisely tuned super-twisting controller having similar specifications as that of the conventional gasoline engine. The resulting approach confirms the significant decline in engine part load losses and improvement in the thermal efficiency and, accordingly, exhibits considerable enhancement in the fuel economy of the VCR Atkinson cycle engine over Otto cycle engine during the wide operating range.

Keywords: Sliding mode (SM) control, Atkinson cycle engine, fuel economy, mean value engine model, variable valve timing (VVT) and variable compression ratio (VCR)

1. INTRODUCTION

The gradual decline of oil reserves, ever growing energy demands along with the stringent emission regulations over the last decades, have led the automotive researchers to seek practical solutions so as to decrease the reliance on fossil fuels for transportation. Engine downsizing has been broadly recognized as an enabler to attain aforesaid objectives Wang and Xu (2012). Atkinson cycle engine is one of the engine downsizing technologies Atkinson (1987) used in hybrid electric vehicles (HEVs) instead of the standard Otto cycle engine.

The efficiency of the spark ignition (SI) engine degrades while working at part loads. It can be optimally dealt with a slightly different thermodynamic cycle termed as an Atkinson cycle. It can be implemented in the conventional SI engines by incorporating advanced mechanism as variable valve timing (VVT) owing to the recent technological developments Pertl (2012). An ideal Atkinson cycle PV representation is shown in Fig. 1 (wherein 6-1-2-3-4-5-6 characterizes the Atkinson cycle, while the conventional Otto cycle is represented by 6-1-2-3-4*-5*-6). The potential advantages of the Atkinson cycle engine as compared to the Otto cycle engine, in terms of the fuel economy improvement and exhaust emissions decline stem from the fact that the standard SI engines that are used in the automotive cars are designed to have optimum efficiency at the full load. Whereas, most of the time the automotive engines have to be operated at part loads (light loads), their load is being controlled through the conventional throttle. As a result, intake manifold pressure drops considerably and, accordingly, pumping losses (throttling effect) increase that leads to substantial inefficiency.

Likewise, the thermal efficiency degrades meaningfully at engine light loads as the conventional SI gasoline engines have fixed compression ratios ranges between 8 to 12 and are limited by fuel quality and knock. Martins and Jasasky (2004) examined the part load concerns of the Otto cycle engine and have derived the thermal efficiency of the conventional Otto cycle engine described as:

$$\eta_{otto} = 1 - \frac{1}{r_c^{\gamma-1}} - \left(\frac{1}{B}\right)\left(1 - \frac{1}{\beta}\right)\left(1 - \frac{1}{r_c}\right) \quad (1)$$

where,

$$B = \frac{\eta_c Q_{LHV}}{RT_m(AFR+1)}, \qquad \beta = \frac{P_m}{P_a}$$

For parameters details see Table 4.1 Murtaza (2017). The research Parvate-Patil and Gordon (2003); Payri et al. (2015); Feng et al. (2016) investigates comprehensively the importance of variable valve actuation (VVA), VCR, the vital role of the accurate engine model and robust

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★ This work was not financially supported by any organization

10.1016/j.ifacol.2017.08.562

controllers on the SI engine's fuel consumption. Moreover, research Guzzella and Onder (2004); Martins and Jasasky (2004); Pertl (2012) examined the thermal efficiencies of several engine cycles and concluded that the Atkinson cycle (over-expanded cycle) is optimum thermal efficiency contributing thermodynamic cycle.

Detailed studies have been accomplished on the over-expanded engine cycle by using cylinder-by-cylinder (CCEM) models, through LIVC load handling approach, theoretically as well as experimentally. About 6% improvement in the fuel economy of the VCR Over-expanded engine cycle using CCEM model over the Otto cycle engine has evaluated during the standard New European Driving Cycle de Sousa Ribeiro (2006). The specific fuel consumption (SFC) of this engine cycle, realized by VVT as well as using VCR mechanism has been improved around 19% over the conventional Otto cycle SI engine, at medium and higher loads Martins and Jasasky (2004). Sugiyama and Aoyama (2007) has reported 16.2% reduction in the fuel consumption as a result of the innovated mechanisms.

Another significant aspect to accomplish the engine performance (fuel economy, emission reduction and vehicle drivability), the availability of accurate and simple control-oriented engine model incorporated with innovative technologies and the engine robust controllers, are the key contributors G. Murtaza and Ahmed (2017); Stefanopoulou (1996). In this connection, Puleston et al. (2001) have used a dynamic sliding mode design approach for speed control of the conventional SI engines, whereas Khan et al. (2001) has employed 2-sliding super-twisting algorithm for the conventional engine speed control based on MVEM model. The SMC approaches, demonstrate robustness of the closed-loop system against parameter variations, changes in the load torque and initial speed. The work Zhang et al. (2010) has also addressed speed control problem of the SI engine by using a proportional feedback controller scheme as well as the stability analysis of the proposed control system by considering its transient performance.

However, most of the prior studies in the assessment of the over-expanded cycle engine, incorporated with the innovative mechanisms regarding fuel economy, have taken into account either the conventional Otto cycle MVEM model or CCEM models of the VVT engines only. Whereas, CCEM have a number of limitations such as, complexity and difficulty in analysis and control design. Design, development and evaluation of any robust control strategy based on MVEM model of the Atkinson cycle engine is not present in the literature.

This research seeks to demonstrate sliding mode strategy, for speed control of the Atkinson cycle engine through an alternative breathing (LIVC) control channel, using super-twisting algorithm. The control framework is based on EMVEM model of the Atkinson cycle engine and is evaluated in the view of fuel economy for the standard FUDS and US06 driving cycles. The simulation results exhibit the considerable decline in engine part load losses, enhancement in the thermal efficiency and, accordingly, confirm improvement in the fuel economy of the VCR Atkinson cycle engine. The organization of the paper is as follows: The description of the control-oriented EMVEM model of an Atkinson cycle engine is presented in Section 2. EMVEM model based SOSM control framework for the Atkinson cycle engine is developed in Section 3. In Section 4, evaluation of the control framework in view of fuel economy is discussed and finally the conclusion is presented.

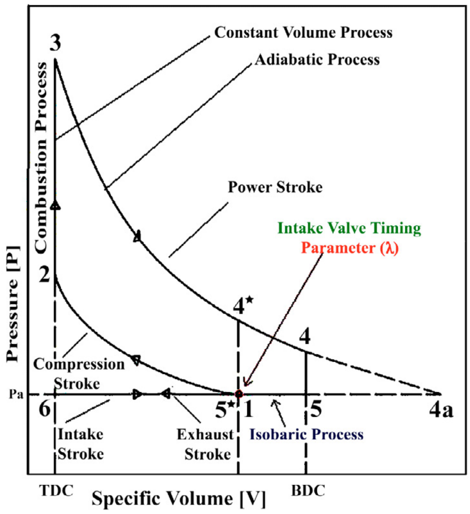

Fig. 1. Theoretical PV depiction of an ideal Atkinson cycle engine Murtaza et al. (2016); Pulkrabek (2003)

2. ATKINSON CYCLE ENGINE MODEL DESCRIPTION

The model accuracy of the SI engine with flexible advanced technologies plays vital role in its performance improvement Guzzella and Onder (2004); Pulkrabek (2003). In this view, the authors Murtaza et al. (2016) have developed a control-oriented EMVEM model of the Atkinson cycle engine, in which the characteristics such as variable valve timing, VCR, over-expansion and realization of the Atkinson cycle have been integrated, is presented in the following subsection.

2.1 CONTROL-ORIENTED EMVEM MODEL

The Atkinson cycle VVT engine's dynamics comprising of the key modeling processes, such as air dynamics which consists of intake manifold pressure dynamics with VVA incorporated to realize the Atkinson cycle, throttle body, air mass suction into the engine cylinders, the rotational dynamics and then the fuel dynamics Murtaza et al. (2016); Hendricks and Sorenson (1990) established on

- Fluid Dynamics Principles
- Thermodynamics Principles
- Atkinson cycle for the in-cylinder dynamics analysis
- Inertial laws

is given as:

$$\begin{cases} \dot{P}_m = \Psi_1 \chi(p) - (2-\lambda)\Psi_2 P_m \omega_e \alpha(P_m, \omega_e) \\ \dot{\omega}_e = \dfrac{1}{J_e}\left(T_{ind}(\lambda) - T_{pump} - T_{fric} - T_{load}\right) \\ \ddot{m}_{ff} = \dfrac{1}{\tau_f}\left(-\dot{m}_{ff} + X_f \dot{m}_{fi}\right) \end{cases} \tag{2}$$

The complete model details, symbols, their description and values are given in dissertation Murtaza (2017). In this model, late intake valve closing load control strategy is employed, instead of the conventional throttle. IVT parameter λ at WOT throttle is considered as a control variable and angular speed as an output while evaluating fuel consumption. In this control frame work any robust control approach can be used. In the following section, however a robust super-twisting algorithm design for Atkinson cycle engine's speed control subsequently used to compute the fuel economy of VCR Atkinson cycle engine over conventional SI engine is presented.

3. CONTROL SYSTEM DESIGN

To design second order mode control for the Atkinson cycle engines based on nonlinear EMVEM model (2) with an alternative breathing control strategy, its design dynamics is described as:

$$\dot{x} = f(x) + g(x, u) \tag{3}$$

$x = [Pm, w_e, \dot{m}_{ff}]^T \in \Re^3; u \in \Re; f : \Re^3 \rightarrow \Re^3; g : \Re^3 \rightarrow \Re^3$; $f(x)$ and $g(x, u)$ being smooth vector fields (see Appendix B Murtaza (2017)). Where u is control input (IVT parameter). As the third term in the expression of η_{atk} in the model, during the engine entire operating conditions (within the constraints on IVT parameter i.e. $1 \leq \lambda \leq 1.60$) have utmost value approximately 0.004, for the computation of the controller design parameters only this term has been ignored for simplicity.

3.1 CONTROL OBJECTIVES

Establishing the control objective and, accordingly, defining the switching surface is the second stage in the control design practice. In this work, evaluation of the potential advantages of the VCR Atkinson cycle engine with variable LIVC load handling strategy in terms of fuel economy over the conventional Otto cycle SI engine at medium and higher load conditions is the proposed objective. The anticipated aim can be achieved by controlling the engine speed through a controller robust against the disturbances, uncertainties in the model parameters and un-modeled system dynamics, which has pivotal significance in the accomplishment of optimum engine fuel consumption as well as vehicle drivability.

In the variable structure control (VSC) theory framework, the switching surface (control objective) can be described as:

$$s(t, x) = \omega_{eref} - \omega_e \tag{4}$$

where ω_{eref} is the engine reference speed and s is the switching variable must steered to zero.

3.2 CONTROLLER DESIGN

The sliding surface s have relative degree one with respective to the control input u (i.e. IVT parameter λ) as u does not appear explicitly in the sliding surface, whereas, it appears in the $\dot{s}$ expression. It is assumed that information about s is available. Consequently, by differentiating the sliding surface twice; the following relations are derived:

$$\dot{s} = \frac{\partial}{\partial t} s(t, x) + \frac{\partial}{\partial x} s(t, x).\left(f(x) + g(x, u)\right) \tag{5}$$

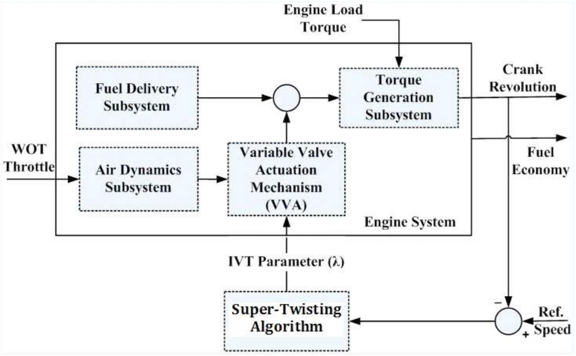

Fig. 2. Schematic diagram of closed loop Atkinson cycle VVT engine system

$$\ddot{s} = \frac{\partial}{\partial t}\dot{s}(t, x) + \frac{\partial}{\partial x}\dot{s}(t, x, u).\left(f(x) + g(x, u)\right) + \frac{\partial}{\partial u}\dot{s}(t, x, u).\dot{u}$$
$$= \varphi(t, x, u) + \xi(t, x, u).\dot{u} \tag{6}$$

The above-mentioned expressions detail are specified in the Appendix B Murtaza (2017).

Afterward, through the EMVEM model analysis, the essential conditions for designing the SOSMC control fulfilled by dynamics (4) are demonstrated, which are given as follows.

- The control signal belong to the $U = \{u : |u| \leq U_m\}$, with $U_m \in \Re$.
- There exist $u_1 \in (0, 1)$ such as for any continuous function u with $|u(t)| > u_1$, there exist $s(t)u(t) > 0$ for $t > t_0$. Therefore, the control effort $u(t) = -sign\left(s(t_0)\right)$, where t_0 is the initial time value, provides hitting the sliding surface $s = 0$ in finite time.
- For the given $\dot{s}$, there exist $+Ve$ constants $s_0, u_0 < 1$, Γ_m, Γ_M, such as if $|s| < s_0$ then
$$0 < \Gamma_m \leq \frac{\partial}{\partial u}\dot{s}(t, x, u) \leq \Gamma_M, u \in U, x \in X. \tag{7}$$
The bounds Γ_m, Γ_M are determined from the detailed analysis and the comprehensive simulation studies of the EMVEM dynamics for medium and higher engine operating conditions for $s_0 = 4.6e^{-}5$ as follows:
$$\Gamma_m = 45.3 \quad \Gamma_M = 70.6$$
- Furthermore, a $+Ve$ constant $\Phi = 4.197$ has been computed, as within the region $|s| < s_0$, for every time $u \in U, x \in X.$, the following inequality holds:

$$\left|\frac{\partial}{\partial t}\dot{s}(t, x) + \frac{\partial}{\partial x}\dot{s}(t, x, u).\left(f(x) + g(x, u)\right)\right| \leq \Phi \tag{8}$$

Thus, with the solutions of the following equivalent differential inclusion by applying SOSMC, the control problem of the system (3) with dynamics (6) can be solved:

$$\ddot{s} \in [-\Phi, \Phi] + [\Gamma_m, \Gamma_M]\,\dot{u}. \tag{9}$$

Finally, the controller parameters can be design, by using the information of the global bounds of $\varphi(.)$ and $\xi(.)$. A schematic illustration of the developed control framework is appreciated in Fig. 2.

Accordingly, a control law based on the super-twisting algorithm, the most popular algorithms among the SOSM algorithms is developed Levant (1993). By establishing, the conditions (7) and (8), the main advantages of this algorithm relies on its robustness to disturbances and parametric uncertainties. As the engine EMVEM model has relative degree one, so the control law can be defined as combination of discontinuous time derivative and continuous function of the sliding surface:

$$u(t) = u_1(t) + u_2(t)$$
$$\dot{u}_1(t) = -\xi sign(s)$$
$$u_2(t) = \begin{cases} -\mu|s_0|^{\rho} sign(s), & \text{if} \quad |s| > |s_0| \\ -\mu|s|^{\rho} sign(s), & \text{if} \quad |s| \leq |s_0| \end{cases} \tag{10}$$

where ξ, μ and ρ are controller design parameters. For finite time convergence to the switching manifold the corresponding sufficient conditions are Levant (1993):

$$\xi > \frac{\Phi}{\Gamma_m}$$
$$\mu^2 \geq \frac{4\Phi}{\Gamma_m^2} \frac{\Gamma_M(\xi + \Phi)}{\Gamma_m(\xi - \Phi)}$$
$$0 < \rho \leq 0.5 \tag{11}$$

By satisfying the above-mentioned conditions, the controller parameters are designed to ensure the establishment of the SOSMC in finite time. Keeping in view to have low content of high frequency components in the control, the super-twisting controller parameters are selected. The most adequate set of controller parameters chosen as a result of an iterative refining procedure are as follows:

$$\xi = 0.09 \qquad \mu = 0.09 \qquad \rho = 0.5$$

In the following subsection, fuel economy of the VCR Atkinson cycle engine over conventional SI engine during wide engine load range is examined. It is to assume that the air to fuel ratio is being controlled/ maintained at stoichiometric ratio by an independent robust controller. At WOT throttle, LIVC (IVT parameter) load control strategy is employed, whereas exhaust valve timings and intake valve opening have been considered unchanged in this work.

4. EVALUATION OF THE CONTROL FRAMEWORK

To study the potential benefits of the VCR Atkinson cycle engine with its intrinsic variable LIVC load handling strategy instead of the conventional throttle, a SOSMC framework designed for the Atkinson cycle engine is evaluated by using its already developed control-oriented EMVEM model. This research is conducted to assess fuel economy of the VCR Atkinson cycle engine over the conventional Otto cycle engine, at the medium and higher load operating conditions, during the standard Federal Urban Driving Schedule (FUDS) and US06 driving cycles. The notion of VCR mechanism is used to cope with the causes of the innovative mechanisms incorporated in the Atkinson cycle VVT engines. To model the VCR effect, the combustion chamber's height is changed at each engine load simulation in order to accomplish around the same maximum in-cylinder temperature and pressure de Sousa Ribeiro (2006). Therefore in the perspective of fuel consumption investigation, both the engine,s part load losses, thermal efficiencies, air mass flow rate, fuel mass flow rate and subsequently fuel consumptions are worked out for the FUDS and US06 driving cycles.

The procedure adapted for evaluation of fuel economy is elaborated comprehensively for FUDS driving cycle.

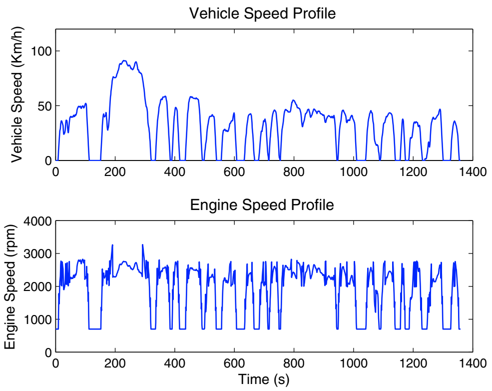

Fig. 3. Vehicle and, accordingly, the engine speed profile for the FUDS Driving Cycle

Generally, vehicle speed and the gear selection define the driving cycle. FUDS chosen to compute the required engine speed at each instant. The parameters used to calculate the engine speed demand during driving cycle corresponding to an advanced car specifications are as described in Table 5.1 Murtaza (2017). The study of fuel consumption at engine level requires engine speed in accordance with the vehicle speed. At every instant of vehicle driving cycle, the engine speed to be considered as a reference is calculated by using the following relation

$$\omega_e = \frac{v_{cyc}.G.g_k}{r} \tag{12}$$

where, g_k represents various gear ratios (i.e. 1^{st} to 5^{th}) depending on the vehicle speed. The vehicle speed and, accordingly, engine speed profiles used as a reference are as shown in Fig.3.

The comprehensive simulation results with engine speed profile as a reference of the closed loop the Atkinson cycle VVT engine, performance degrading aspect and various factors having vibrant role in the enhancement of fuel economy at 12Nm load conditions are elaborated here. Finally, compact findings regarding the fuel consumption at different operating conditions are presented. Fig. 4 reveals speed tracking profile for FUDS driving cycle. Super-twisting controller exhibits good reference tracking. Fig. 5 appreciated the typical twisting effect imposed by super-twisting controller. It guaranteed the stability

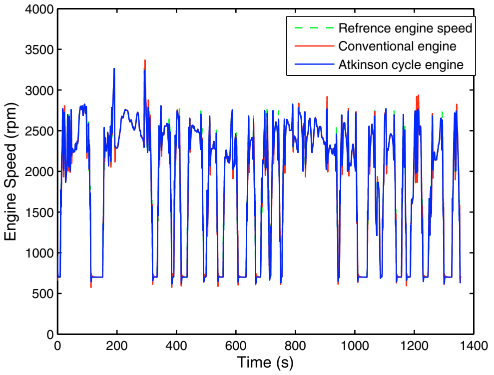

Fig. 4. Speed tracking profile using the super-twisting algorithm at 12Nm load

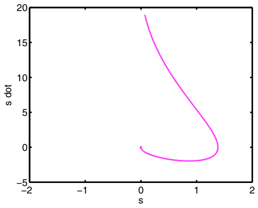

Fig. 5. s(t) versus $\dot{s}(t)$

of the closed loop system, as the differential inclusion (11) is fulfilled. The evolution of the control signal (IVT parameter λ) and switching surface throughout the driving cycle at 12Nm operating load is as divulge in Fig. 6. The control signal variations confirm the fact that intake valve closing timing is primarily VVT engine operating conditions dependent.

A comparative view of the Atkinson cycle engine's pumping losses with that of the conventional Otto cycle during the complete FUDS driving cycle at 12Nm load, is illustrated in Fig. 7. It can be seen that Atkinson cycle engine part load losses are comparable to WOT losses of the conventional engine. The reduction in pumping losses with LIVC strategy is predominantly the result of back flow of charge into the pressure manifold or it simultaneously turns into the intake of another combustion cylinder. Thus, suction pressure rests considerably above the atmospheric pressure during the first part of compression stroke, which is a key contributor in lessening of pumping losses. Fig. 7 also demonstrates the thermal efficiency of the VCR Atkinson cycle engine in association with that of the conventional SI engine throughout FUDS. A considerable enhancement in the EMVEM model's thermal efficiency can be seen, as engine load decreases further down the full load. With LIVC load control approach around 3.52% improvement in the thermal efficiency is achieved at 12Nm load conditions within constraints of IVT parameter.

The comparison of fuel mass flow rate (instantaneous fuel consumption) of VCR Atkinson cycle engine as well as the conventional Otto cycle engine under similar operating conditions during the FUDS and US06 is presented in the Fig. 8. By integrating the fuel mass flow rate for the complete FUDS cycle the total fuel consumed during the driving cycle can be obtained and 6.53% improvement in fuel economy is achieved for this particular case.

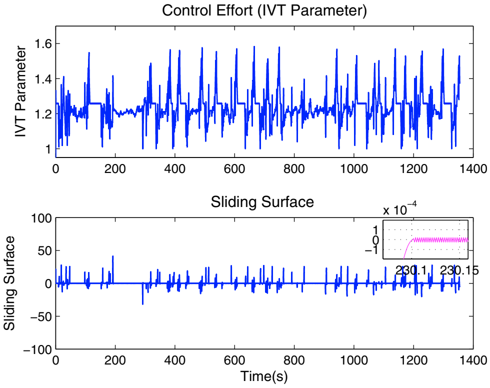

Fig. 6. Control effort (IVT parameter) and switching surface during the FUDS at 12Nm load conditions

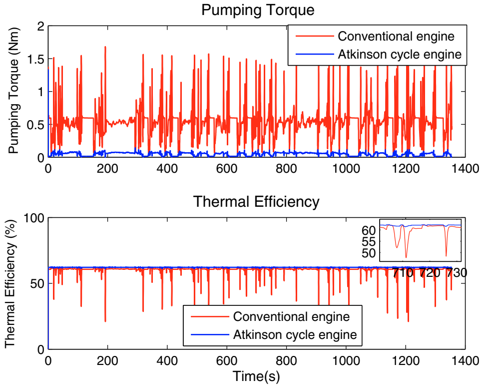

Fig. 7. Pumping losses and thermal efficiency comparison during FUDS at 12Nm load conditions

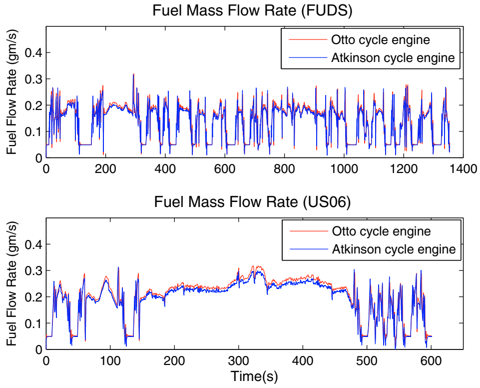

Fig. 8. Fuel mass flow rate comparison during FUDS and US06 at 12Nm load

The fuel consumption comparison accomplished with VCR Atkinson cycle engine over the conventional Otto cycle engine at various engine operating conditions, for instance at 8Nm, 10Nm, 12Nm and 14Nm during the standard FUDS driving cycle. It can be seen that improvement in the fuel economy of the Atkinson cycle engine goes on increasing towards the part load conditions in contrast with the conventional Otto cycle engine. The enhancement in fuel economy at higher load i.e. 14Nm load is 4.57%, whereas augmentation in fuel economy at part load i.e. 8Nm is around 11.50%. However, on the average around 4.32% enhancement in the thermal efficiency, while about 7.80% improvement in the fuel consumption with VCR Atkinson cycle engine over the conventional Otto cycle engine at medium and higher engine operating conditions during the FUDS is attained. Likewise, fuel consumption of the VCR Atkinson cycle engine in comparison with the conventional engine under similar engine operating conditions during the US06 driving cycle is evaluated.

The results comprising of Atkinson cycle VCR engine operating conditions, thermal efficiency and fuel consumption improvements for FUDS and SU06 driving cycles, are summarized in Table 3. The simulation outcome reveals that on the average about 4.32% enhancement in the thermal efficiency and around 7.80% improvement in the fuel consumption with VCR Atkinson cycle engine over the conventional engine at medium and higher load operating conditions during the FUDS are accomplished. To avoid

Table 1. Simulation results summary

		FUDS		**US06**	
Engine operating conditions	**Compression ratio**	**Thermal efficiency enhanc. (%)**	**Fuel economy improv. (%)**	**Thermal efficiency enhanc. (%)**	**Fuel economy improv. (%)**
8Nm	14.65	7.20	11.50	4.59	10.23
10Nm	14.35	4.67	8.62	3.56	8.46
12Nm	13.80	3.52	6.53	2.89	6.19
14Nm	13.15	1.88	4.57	1.66	4.46
Average improvement		**4.32**	**7.80**	**3.18**	**7.33**

the effect of LIVC on ignition delay and engine combustion duration, LIVC can be physically realized for medium and higher loads. Below 40% of the full engine load, the use of throttle may be required as a substitute, without losing the potential benefits of the integrated VVT system de Sousa Ribeiro (2006).

5. CONCLUSION

This research outlines the design and development of sliding mode control framework for the Atkinson cycle engine with an unconventional flexible LIVC load control strategy. The control structure based on physically motivated control-oriented EMVEM model of the VCR Atkinson cycle engine is evaluated for FUDS and US06 driving cycles at medium and higher operating conditions to ensure its better fuel economy in comparison with the conventional SI engine. A robust SOSM strategy based on super-twisting algorithm is used to resolve the VVT engine speed control problem avoiding chattering. The simulation results exhibit the suitability of SOSM algorithm to control Atkinson cycle engine with an alternative input channel. The part load losses of the Atkinson cycle engine figured out are comparable to the WOT losses of the conventional SI engine, whereas a considerable boost in the thermal efficiency is accomplished during FUDS and US06. The aforesaid improvements play pivotal role in reducing the VVT engine's fuel consumption. On the average, around 7.80% and 7.33% of the fuel economy with VCR Atkinson cycle engine over the conventional SI engine during the FUDS and US06 driving cycles, is achieved correspondingly.

ACKNOWLEDGEMENTS

The author's acknowledgment is extended to the Center for Automotive Research (CAR) at The Ohio State University, Columbus, USA; for providing the technical guidance for this work.

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